On June 8, 2004 you may be able to see the planet Venus as it moves across the face of the sun. (see viewing map below)

2004 Transit of Venus

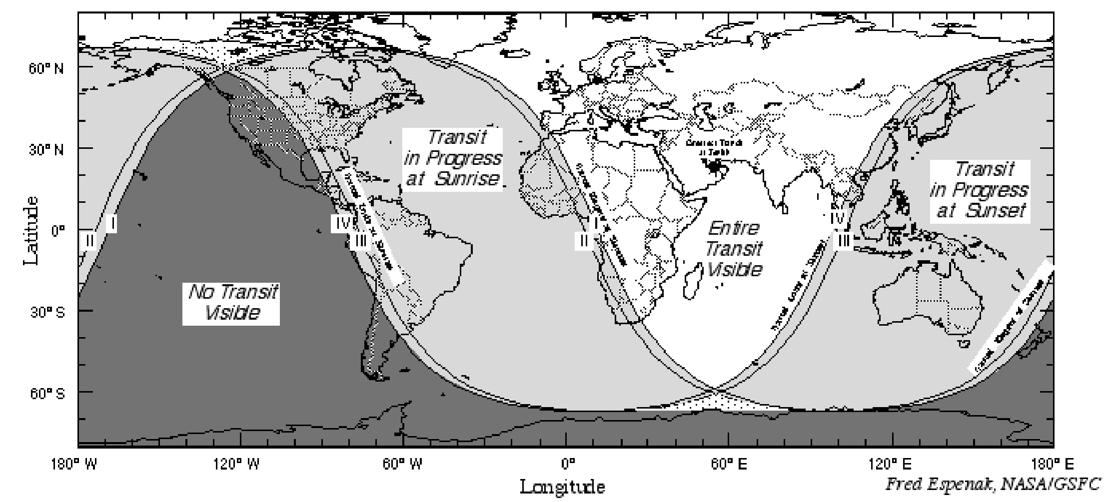


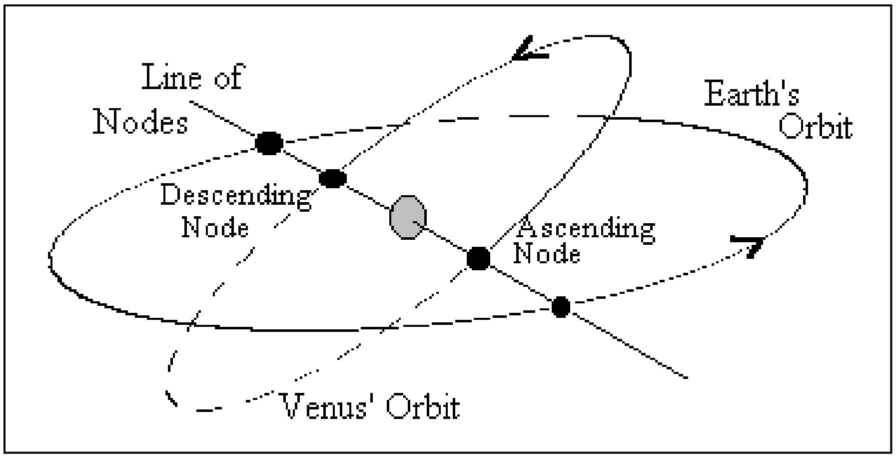
Figure 2 - World Visibility of the Transit of Venus — 2004 June 08

Courtesy Fred Espenak, NASA Goddard Space Flight Center

The last time humans witnessed this event was on December 6, 1882 when it was watched by millions of people across the world, from the crowded streets of Bombay to the deserts of the American southwest.

The first time on record was only 243 years before that in 1639. Johannes Kepler correctly predicted that Venus would pass in front of the Sun in December 1631, but no-one observed it - because it occurred after sunset for most of Europe. Kepler also predicted that Mercury would pass in front of the Sun. Pierre Gassendi observed this transit in Paris a month before the predicted Venus transit. Since Venus and Mercury are so far from Earth, they appear as small dots against the Sun. Therefore, observations of transits were only possible using a telescope - an instrument introduced to astronomy around 1610 by Galileo Galilei. Using Kepler's methods Jeremiah Horrocks managed to predict that a further transit of Venus would occur on December 4th ,1639, 8 years after the one predicted by Kepler. Using a simple telescope to project an image of the Sun on paper, Horrocks was able to observe part of the 1639 transit and become the first human on record to observe this intriguing and rare event.

There is a curious 243 year repeating pattern with two transits in December (around the 8th), eight years apart, then a wait of 121 and a half years, then two June transits (around the 7th), again eight years apart, then a wait of 105 and a half years and then the pattern repeats again. This occurs because Venus' orbit is tilted when compared with Earth's orbit.

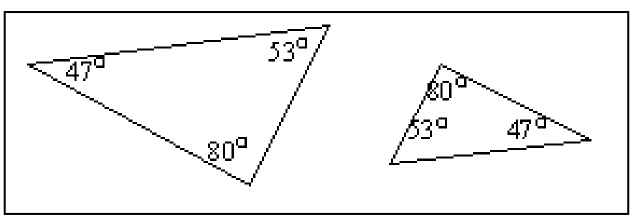


The Transit of Venus may be a rare event, but it has proved important to early calculations of the distance from Earth to the Sun. Knowing the distance from the Sun to Earth allowed astronomers to calculate distances to all of the other planets. Before the critical measurements of the Transit of Venus in the late 1800s, distances in the solar system were expressed in Astronomical Units (AU). But nobody knew what an AU equaled in terms of miles or kilometers. The AU was simply the distance from Earth to the Sun; all distances from the other planets to the Sun were calculated using Kepler's Laws in comparison with the Earth-Sun distance. So astronomers needed to calculate the AU in kilometers!

To do this calculation for yourself, you need to understand a couple of simple ideas from geometry.

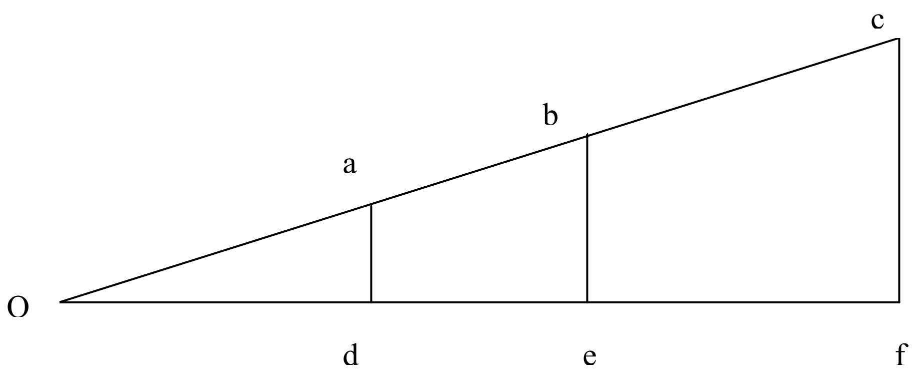
Math Excursion

The first idea has to do with **similar triangles**. A characteristic of similar triangles is that all three angles of one triangle are equal to the three angles of the other triangle. The two triangles below are similar.



If two triangles are similar then the length of their sides and altitudes are **proportional**. That means that if, for example, one side of the larger triangle is three times as long as the corresponding side (across from the same size angle) of the smaller triangle, then all the sides of the larger triangle are three times as large of their corresponding parts in the smaller triangle.

We are going to look at very special similar triangles – triangles with one 90° (called right triangles).



In the figure above, the three vertical lines create three similar triangles – Δaod, Δboe, and Δcof.. Sides **da**, **eb** and **fc** are called corresponding sides because they are all opposite the same angle. Sides **ao**, **bo**, and **co** are also corresponding sides, but they have a special name because they are opposite the 90° angle – they are called the hypotenuse of their triangles. Can you name other corresponding sides?

(The three vertical lines were placed randomly. Placing the vertical lines in different locations could make an infinite number of similar triangles.)

A very interesting thing is true about the lengths of the sides of these triangles because they are similar triangles.

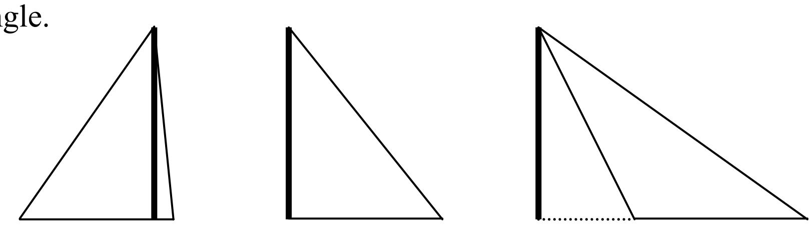
> Side **da** divided by side **do** is equal to side **eb** divided by side **eo** and is equal to side **fc** divided by side **fo**.

$$\frac{\mathbf{da}}{\mathbf{do}} = \frac{\mathbf{eb}}{\mathbf{eo}} = \frac{\mathbf{fc}}{\mathbf{fo}}$$

Measure these sides yourself and do the division to convince yourself that this is correct.

The lines **do**, **eo**, and **fo** are altitudes of the three similar triangles.

Definition of altitude: The altitude is the line from one vertex (or point) of the triangle to the opposite side – and is perpendicular to the opposite side. The dark line in the drawings below is the altitude of each triangle. Note that the altitude in the right triangle is outside the triangle.



So we can say that the base divided by the altitude of one triangle is equal to the base divided by the altitude of a <u>similar</u> triangle.

Notice that **da**, **eb**, and **fc** are all opposite angle O, and **do**, **eo**, **fo** are next to or adjacent to angle O. This brings us to a second idea that we will use.

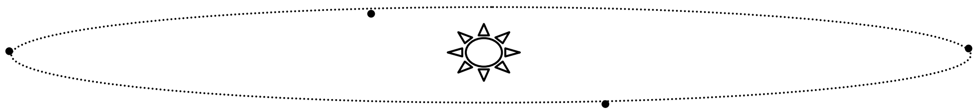
For any angle in a right triangle (except the 90˚ angle), the side opposite the angle divided by the side adjacent the angle always gives the same number. Draw other vertical lines in the figure above and test this yourself.

Since this is always true for right triangles, this relationship is given a special name. It is called the **tangent**. For example, if angle O were a 30˚ angle, the side opposite/side adjacent would always equal 0.57735 (rounded to 5 decimal places). This is called the tangent of 30˚ (or tan 30˚). You can try this on your calculator.

The relationships between other sides of a right triangle also have special names. The side opposite divided by the hypotenuse is called the **sine** of the angle (example, sin 30˚ = .5). The side adjacent divided by the hypotenuse is called the cosine (example, cos 30˚ = .866). There are three other relationships, but the tangent and sine will be enough for us.

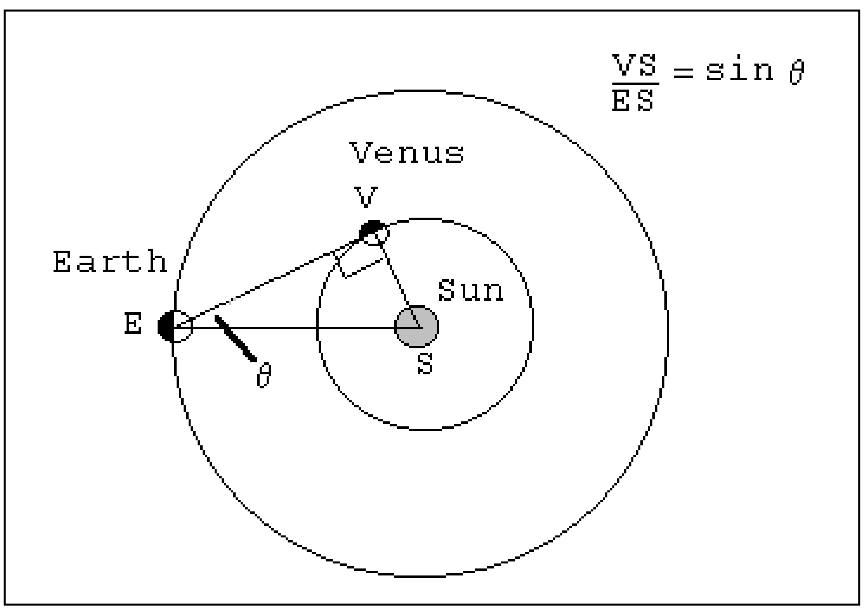
Back to Venus

What does this have to do with Venus? When astronomers accepted that the planets orbit the Sun, it was pretty clear from observations of Venus that is was closer to the Sun than Earth is. The diagram below shows the Sun and Venus in its orbit (**not to scale**) as it might look from Earth.



How Far Is Venus From the Sun?

Venus could appear close to the Sun or far, and it might be to the left or right as we observed it from Earth. When Venus it at its farthest from the Sun (called maximum elongation), astronomers can calculate how far Venus is from the Sun. If we could look at the Solar System from above the orbits of the planets, we would see the Sun, Earth and Venus as in the diagram below (**not to scale**)



Notice that lines connecting Earth, Venus and the Sun make a right triangle. Astronomers (on Earth, of course) measured the angle θ (theta) and found out it was approximately 46.054 degrees. The side opposite θ is the distance from Venus to the Sun (VS) and the hypotenuse is the distance from Earth to the Sun (ES). That means that the

$$\sin\theta = VS/ES.$$

Since astronomers didn't know the distance from the Earth to the Sun in miles or kilometers (remember – that is what we are trying to figure out), they agreed to a made up unit of distance called the Astronomical Unit (AU). They also decided that Earth to Sun (ES) is 1 AU.

So, $\sin\theta = \dfrac{VS}{1AU}$

$\sin(46.054°) = \dfrac{VS}{1AU}$

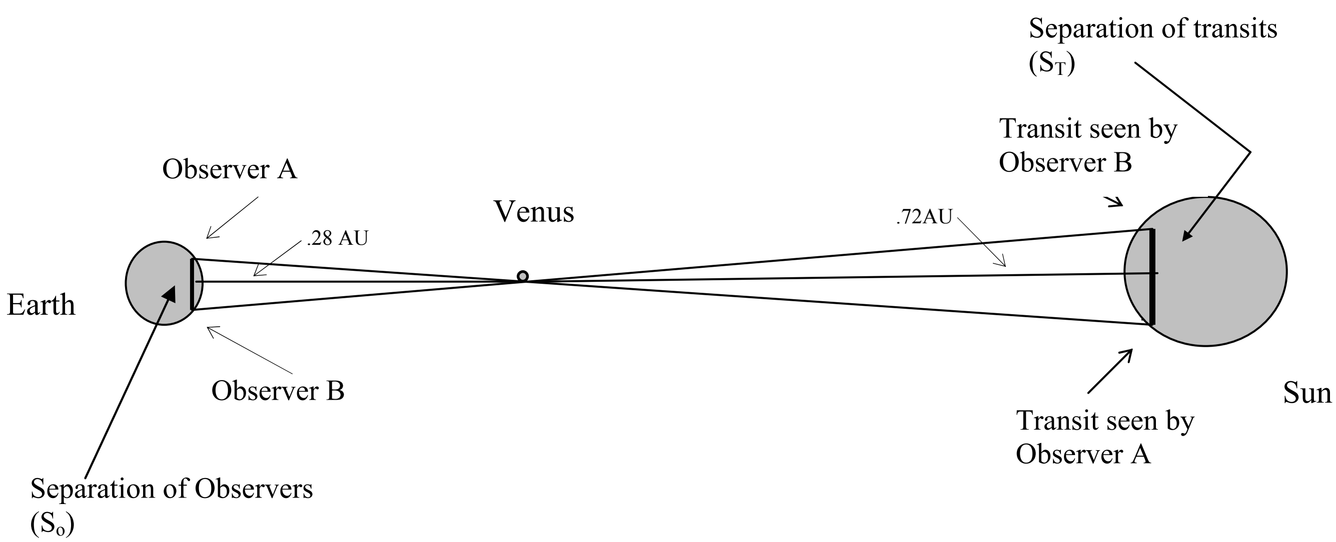
VS was calculated to be 0.72 AU.

So now we know the distance from Venus to the Sun. Sort of! At least we would if we knew what an AU equaled in terms of miles or kilometers.

And we also know that the distance from Earth to Venus is 0.28AU. (1 AU minus 0.72 AU)

Transit of Venus

Now all we need is a transit of Venus and two observers at different latitudes on Earth. Look at the figure below to see how the transit might look (**not to scale and transit separation is greatly exaggerated**).



Observer A at a location in the northern part of Earth will see Venus move across the Sun. Observer B at a location in the southern part of Earth will also see Venus move across the Sun. But Observer A will see Venus cross the Sun on a path lower than what Observer B sees.

> This is called parallax and you can see the effects of parallax yourself. Hold your thumb up at arms length. Close the left eye and block some small object on a far wall with your thumb. Now close the right eye and open the left. Is your thumb still blocking the small object? No! It appears that your thumb is in a different place.

In the drawing above, you should be able to see two similar triangles. One starts at Venus with its base (S_T) on the Sun. The other also starts at Venus and has its base (S_O) on the Earth.

- The altitude of the big triangle is the distances from Venus to the Sun (0.72AU).
- The altitude of the small triangle is the Earth-Venus distance (0.28AU).
- Remember! The base divided by the altitude of one triangle is equal to the base divided by the altitude of a <u>similar</u> triangle.
- Therefore, we can divide the base of the big triangle (S_T) by the altitude (0.72AU) and that will be equal to the base of the small triangle (S_O) divided by the altitude (0.28 AU).

$$\frac{S_O}{0.28\ \text{AU}} = \frac{S_T}{0.72\text{AU}}$$

- Since we can know the separation of the observers (let us assume they are 2000 km apart – say Buffalo, New York and Miami, Florida), we can solve for S_T, the separation of the transit paths as seen from two locations.
- In this case S_T is 5142.86 km.

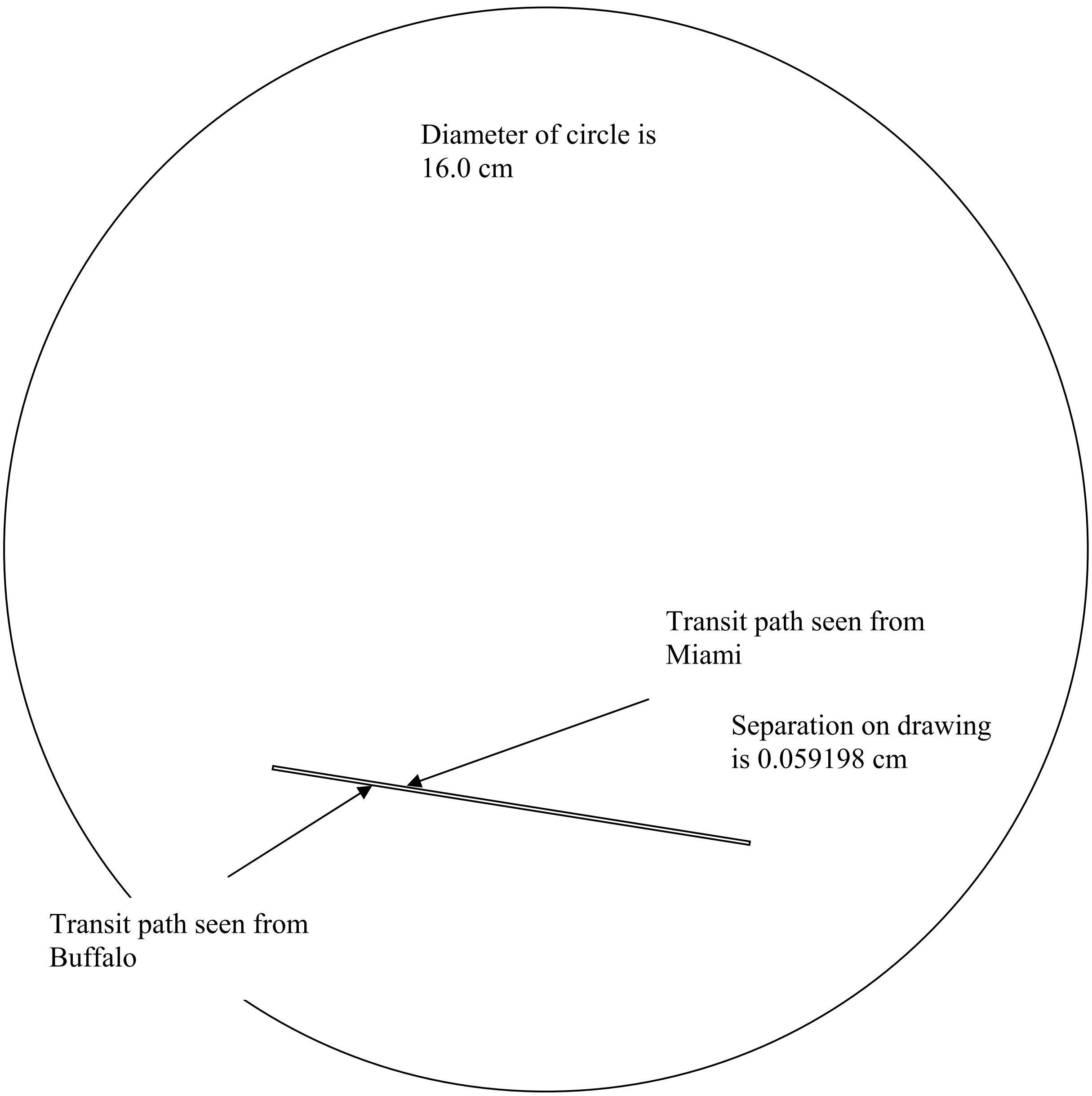
$$\frac{S_O}{0.28\text{AU}} = \frac{S_T}{0.72\text{AU}}$$

Multiply both sides of the equation by 0.72 AU

$$\frac{0.72\cancel{\text{AU}} \times 2000\text{km}}{0.28\cancel{\text{AU}}} = S_T$$

Cancel units and calculate

If you observed the transit of Venus and carefully traced the path of the center of Venus across the Sun, and if an observer 2000km north or south of you did the same, you could lay one drawing on top of the other and see the separation. The figure below is approximately what you would see. The two transit paths are in almost the same place – the two lines are only separated by 0.059198 cm.



This drawing is very much like a map. The distance between the transit lines compared with the diameter of the circle should be the same as the transit separation (S_T) we just calculated compared with the real diameter of the Sun. We can use this comparison to calculate the real diameter of the Sun in kilometers.

$$\frac{\text{Circle diameter}}{\text{Drawing separation}} = \frac{\text{Sun diameter}}{\text{Transit separation } (S_T)}$$

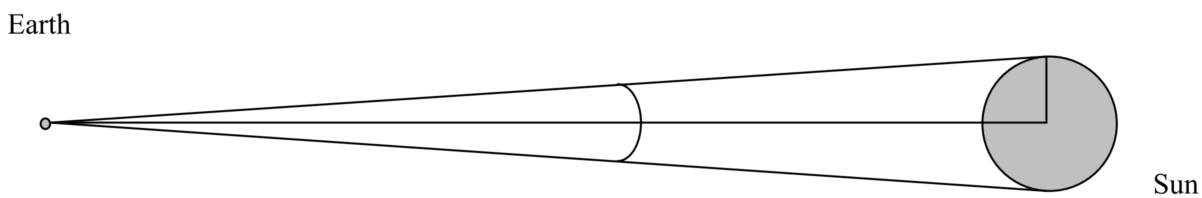
$$\frac{16.0 \text{ cm}}{0.059198 \text{ cm}} = \frac{\text{Sun diameter}}{5142.86 \text{ km}}$$

From this calculation the diameter of the Sun equals 1,390,000 km.

Wait, that is important!!! Now we know the diameter of the Sun! Astronomers didn't know that information before the calculations of Venus Transit!

The Astronomical Unit

Now astronomers can calculate the distance from the Earth to the Sun. Look at the following diagram and think about the tangent definition we had earlier. (diagram not to scale)



When astronomers observe the Sun, they can measure the angle between the line that just 'touches' the top of the Sun and the line that just 'touches' the bottom of the Sun. This angle has been measured many times and the average angle is 0.534˚. But if you look closely at the triangle that is created by the Earth, the center of the Sun and the top of the Sun, you can see this is a right triangle. The very small angle is half of 0.534˚. The side opposite that angle is the radius of the Sun (half the diameter). The distance from the Earth to the Sun is the side adjacent to the small angle.

$$\text{Angle} = \frac{0.534^\circ}{2} = 0.267^\circ$$

Side opposite = 1,390,000 km/ 2 = 695,000km
Side adjacent = 1 AU

tan (0.267˚) = 695,000km/1 AU

therefore, tan (0.267˚) * 1 AU = 695,000 km (multiply both sides of equation by 1 AU)

and , 1 AU = 695,000 km / tan (0.267˚) (divide both sides of equation by tan (0.267˚))

1 AU = 149,140,000 km WOW! Now we know the distance from the Earth to the Sun in kilometers! And we know the value of the AU in kilometers, so we can calculate all other distances in the solar system in kilometers.

For example, we know the distance from Venus to the Sun.
0.72 * 149,140,000 = 107,380,800 km

Now it is time for you to practice so that you can participate in the June 8, 2008 Venus Transit and calculate to Sun-Earth distance.

Imagine that you observe the transit of Venus in Buffalo, New York and a student in Panama City, Panama also observes the transit. Panama City is 3748 km due south of Buffalo. When you compare your drawings of the transit, you get the following drawing. Calculate the Sun-Earth distance (the value of the AU)